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Genetic Algorithms with Division and Integration of Region for TSP

Toshihiko Ono and Takeshi Ikeda
Dept. of Computer Science and Engineering
Fukuoka Institute of Technology
3-30-1 Wajiro-higashi, Higashi-ku, Fukuoka 811-02, Japan
e-mail: ono@cs.fit.ac.jp

Abstract

In this paper, a new method of genetic algorithms (GAs) to decrease the calculation time of the traveling salesperson problem (TSP) is described. Although various methods have been studied to solve the TSP, the GAs seem to be most promising. The TSP is a NP-hard combinatorial problem; therefore, as the number of cities increases, it becomes extremely difficult to get the optimal solution even by the GAs. The GAs proposed here adopt a divide-and-conquer philosophy. Based on a locality hypothesis, the whole region of cities is divided into a number of small regions, where the GAs are applied to each of these small groups of the cities. After the optimal routes for these divided regions are obtained, the whole route is gained by applying again the GAs to the populations consisting of the routes integrated from the optimal ones in the divided regions. Special considerations are paid to the methods of division and integration. The results of simulation studies showed expected performance.

1 Introduction

The traveling salesperson problem (TSP) is one of the classic problems, which date back to 1932 [5], and its algorithms have been studied by many researchers using various methods, such as dynamic programming, neural networks, genetic algorithms, and so forth. Since the TSP can be used to solve various practical problems, for example, how to determine the order of drilling holes in a printed circuit board to achieve the maximum efficiency, it is still being investigated enthusiastically. It has already been proven that the TSP is NP-hard [5], and getting an answer to it becomes increasingly difficult as the number of cities increases. In solving the TSP, it is more important to get a near optimal solution, within a short time, than to get the optimal one after a long calculation, from the practical point of view. For this requirement, the GAs method is one of the most promising ones.

In our papers, we are proposing a new method to decrease the calculation time necessary for the GAs to the TSP. Since the number of possible routes in the TSP of n cities is given by the equation $\frac{1}{2}(n-1)!$, if it is possible to constitute the optimal route of all the cities by integrating each optimal route gained in divided regions, such as four regions of one fourth the original region, the total required calculation time will considerably decrease. We have constructed our TSP algorithms based on this fact.

2 Principle

Since the number of possible routes in the TSP of n cities is given by the equation $\frac{1}{2}(n-1)!$ as explained in the previous section, it will become much more difficult to get the optimal route as the number of cities increases. On the other hand, this suggests the possibility of making the calculation time decrease by dividing the group of cities into smaller ones.

Let's consider when the region is divided into four sub-regions as shown in Figure 1. If an equal number of cities for each divided region is assumed, the possible number of routes for each divided region is given by the equation $\frac{1}{2}(\frac{1}{4}n-1)!$, and this makes the range of search narrow proportionally in the GAs. As a result, this will bring about considerable reduction of the time required for getting the optimal route by the GAs. To adopt this method, however, we have to investigate the following two matters :

1) does a locality hypothesis hold ?

2) how to divide a region and integrate sub-routes in the divided regions into a whole route.

2.1 Locality hypothesis

As a basis of adopting the above method, " locality hypothesis " should hold. The locality hypothesis here defined requests that in getting the optimal solution in a region, the optimal solution obtained in a small part of the region should be a part of the optimal solution of the original region, that is, in the TSP the optimal route of the cities in some small area should constitute a part of the optimal route of all the cities in the original region. Actually, in the TSP this locality hypothesis seems to hold, because two cities separated from each other by a large distance seem almost impossible to become a neighboring pair in the optimal route, in other words, the neighboring cities in the optimal route should be close in distance to each other. Of course, since there might be some cities which don't follow this hypothesis, such as those near to the boundaries between neighboring sub-regions, some measures should be taken into cosideration, as explained later.

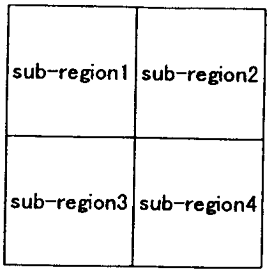

Figure 1: Division of region

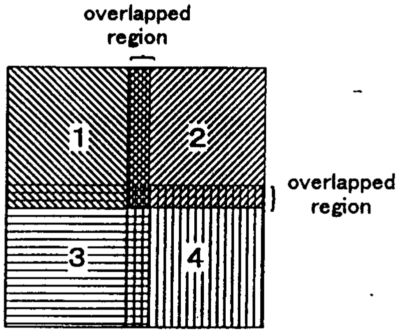

Figure 2: Method of division

2.2 Method of division and integration

There are some regions where the locality hypothesis doesn't seem to hold. They are those near to the boundaries of divided regions, because some cities in those regions may be connected across the boundaries in the optimal route of the whole region. Considering this possibility, we divide the region with the parts that overlap each other, as shown in Figure 2. After the optimal route in each of the divided regions is obtained by the GAs, these four routes are successively integrated to make a whole route in the original region, as follows. First the optimal routes in the regions 1 and 2 are connected to make a route, second the ones in the regions 3 and 4 are connected, and finally these two integrated routes are again connected to constitute a whole traveling route. One of the important problems to solve here is how to deal with the overlapped cities in the integration. Since there is no proper method to get the optimal route for those cities in the overlapped regions, we have constituted many kinds of the integrated routes by various patterns of connections to those cities. By applying the GAs to the initial populations consisting of these integrated routes, we can get the final route. The details will be explained later.

3 System Configuration

The whole system is shown in Figure 3 and operate as follows. First, the original region of cities is divided into four sub-regions with proper overlapped parts. The degree of overlapping is a design parameter. Second, by applying the GAs to each of these sub-regions, the optimal route in each sub-region is gained. Third, the four optimal routes for the sub-regions are combined to make a whole route by connecting the routes 1 and 2, and the routes 3 and 4 respectively, and finally connecting these two integrated routes into one whole route. In the integration, overlapped cities are taken out, and at the same time various patterns of connection are made to the overlapped cities with the aid of random numbers. Fourth, using the thus gained

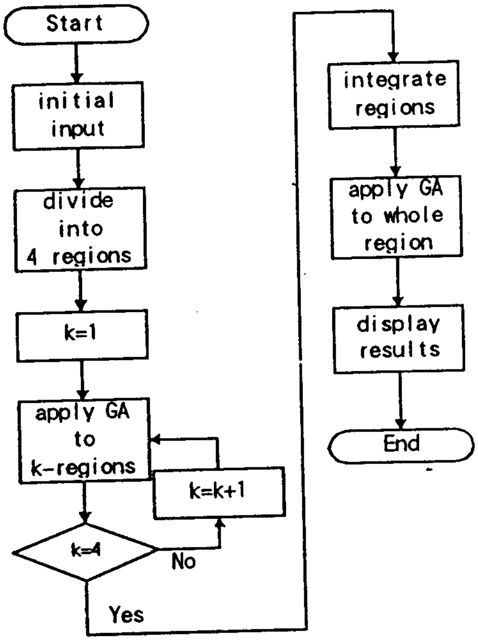

Figure 3: Flow chart of operation

integrated routes of various patterns as initial populations, the GAs determine the final optimal route.

4 Integration Algorithms

By the integration algorithms two sub-routes are combined to make a combined route, while each one of overlapped cities is removed. In this operation, to give the combined route many varieties of path to the overlapped parts, every possible connections for these parts are made. The details of processing will be explained by examples. In the simplest case, take the case where only one city is overlapped and both of the cities neighboring to this city in the route are out of the overlapped region, as shown in Figure 4. When we have reached the overlapped city P after having traversed either one of two routes to be connected - indicated by an arrow in the figure, the next city to proceed to is selected randomly from among three cities Q_2, Q_3, Q_4. Thus six different types of connections are possible, as shown in Figure 5. In the case where more than one city is overlapped, the above procedures are applied repeatedly. Figure 6 shows the case where two successive cities are overlapped.

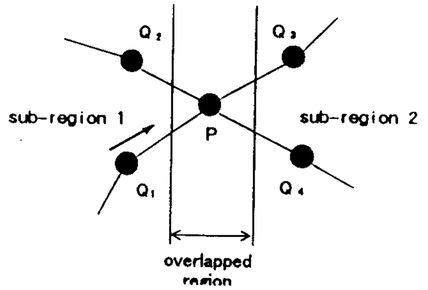

Figure 4: Overlapping of one city

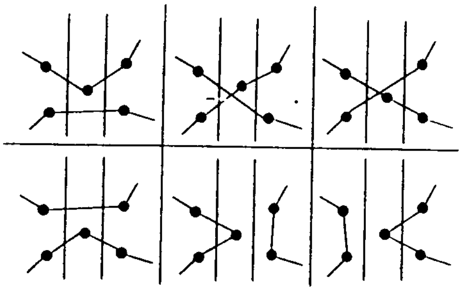

Figure 5: Pattern of connection with one city overlapped

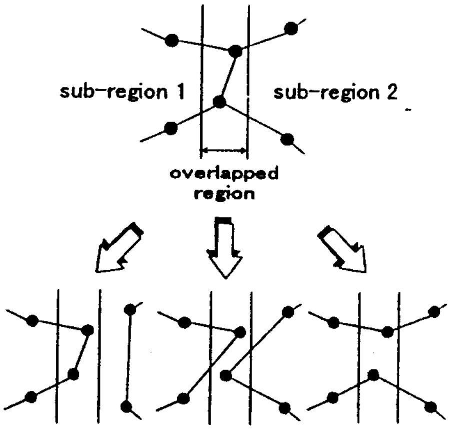

Figure 6: Pattern of connection with two cities overlapped

In the integration of two routes, we have to proceed to one of the two routes forwards or backwards, that is, we have to take either one of the following four passages: forward and backward proceeding on both route 1 and 2. To realize these operations in the algorithms, we have adopted a bi-directional list structure. After two optimal routes of neighboring regions are gained by the GAs, the routes are copied into two bi-directional lists and the integration is done using this list.

The whole process of integration of two neighboring routes, along with the method of connection of overlapped cities will be explained, by referring to Figure 7. Having started at a city in one of the overlapped regions, we proceed to one of the two routes until we arrive at a city in the overlapped region, while copying the passed city on the list of the integrated route and removing it from the list of the proceeding route. At the city in the overlapped region, the next route to take is selected from three

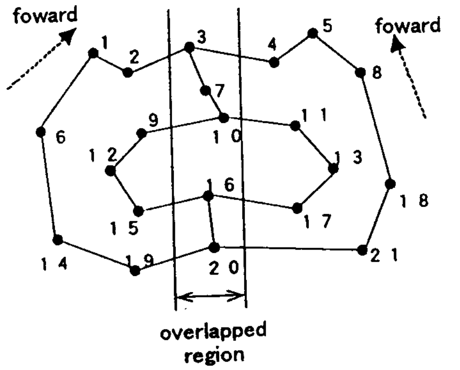

Figure 7: Integration of two routes

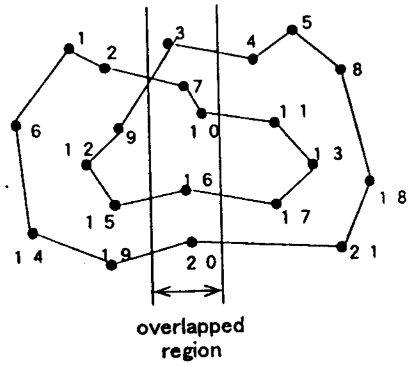

Figure 8: Integrated route

passages excluding that of going backwards. These procedures are repeated until the original two lists become vacant. The route thus gained is shown in Figure 8.

5 Results of Simulation Studies

To test the performance of the proposed method, we have conducted simulation studies and compared the obtained results with those gained by the ordinary method, that is, the GAs without division. All the conditions of simulation for both the proposed method and the ordinary one are set the same. As a software workbench of the GAs, the GENITOR [5]is used. The route in the GAs is represented by the Pass Representation method [5]. The crossover is done by the Edge Recombination method [5]; as for the mutation, the insert mutation is applied, in which a city is randomly picked up and is inserted into an also randomly selected position. As test data, 101 city configuration is used. The number of populations is selected as 500. The GAs operate to make the object value minimum, which is the total tour distance. The number of generations to undergo the genetic operations is set to 40 throughout the simulations.

Figure 9 shows the comparison of convergence characteristics of the proposed method and the ordinary one. As for the proposed method, only the performance after the integration is done, is shown. Since in the proposed method the genetic operations are applied to the divided regions, which sum up 160 generations as a total, the starting point of the genetic operation to the integrated region is equivalent to the 161st generation of the ordinary method. We have made the comparison graph of characteristics, considering these conditions,.

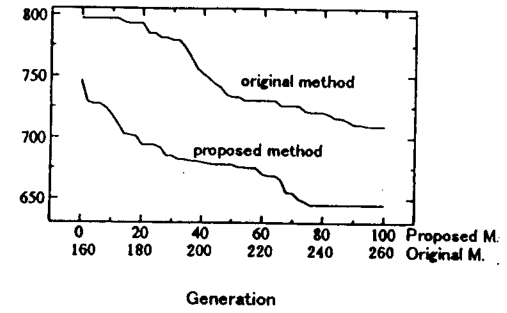

Figure 9: Comparison of convergence characteristics

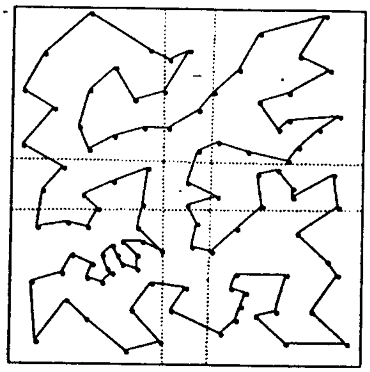

Figure 10: Finally obtained route for 101 cities

From the object value of the proposed method at the starting point after the integration, we may conclude that the locality hypothesis holds. The convergence characteristic of the proposed method is superior to the one obtained by the ordinary method. In addition, the calculation time for the proposed method is much less than that of the ordinary one, because the added time for the division and integration operation in the proposed method is sufficiently compensated by the decrease in the calculation time for the genetic operation of the divided region, brought about by the chromosome length shortened to one fourth the original. Taking these various matters into consideration, we can conclude that the decrease in the calculation time has been achieved by our proposed method. Figure 10 shows the finally obtained route.

6 Discussion and Further Work

We have explained how our proposed method can attain the decrease of the calculation time for the GAs, by referring to the results of simulation. The point of our method is how to divide the region and how to integrate the routes in the divided regions. Special care is paid to the overlapped regions to make various patterns of connection. In regard to the handling of the cities near the boundary, we can adopt various alternative methods other than the explained one.

In the previous explanation, only the division to four sub-regions are performed. This method, however, can be applied recursively many times to get narrower regions, such as 1/16, 1/64, and so on. From our experience the optimal route can be easily obtained for around 30 cities; therefore we will be able to get the optimal or near optimal route for the large TSP easily, through applying our method recursively by dividing until this size.

Furthermore, in this paper the division into four sub-region is explained. There is another option, however, in which a combination of two kinds of divisions to two sub-regions, that is, a horizontal division and a vertical one are applied; the latter division also can be realized by a combination of 90 degrees rotation, horizontal division and reverse 90 degrees rotation. Thus, the algorithms become simpler.

7 Conclusions

We have explained our new high speed calculation method for the large traveling salesperson problem, based on the divide-and-conquer principle. The initial region is divided into small regions, the genetic operations are applied to these narrowed regions, and the optimal route is finally obtained by applying the genetic operations to the integrated route from the optimal routes in the divided regions. Through the simulation studies, the effectiveness of this method has been shown. This method can be applied to not only the traveling salesperson problem, but also various problems, where the locality hypothesis holds, resulting in the decrease of the necessary calculation time, and is especially effective for the large problems.

References

[1] Bäck, T. Evolutionary Algorithms in Theory and Practice, Oxford University Press, 314p, 1996

[2] Chambers, L. Ed. Practical Handbook of Genetic Algorithms, Applications Volume 1, CRC Press, 555p, 1995

[3] Davis, L. Ed. Handbook of Genetic Algorithms, Van Nostrand Reinhold, 385p, 1991

[4] Goldberg, D. E. Genetic Algorithms in Search, Optimization, and Machine Learning, Addison-Wesley Pub. Co., 412p, 1989

[5] Michalewicz, Z. Genetic Algorithms + Data Structures = Evolution Programs, Second, Extended Edition, Springer-Verlag, 340p, 1994

[6] Ono, T. and Ikeda, T. Speed-up of Genetic Algorithms by Division and Integration of Region (in Japanese), Proc. of the Electronics, Information and Systems Conf., I.E.E. of Japan, pp.407-408, 1996