Optimization of Two-dimensional Guillotine Cutting by Genetic Algorithms

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Abstract

This paper presents a new method of optimizing the layout of rectangular parts on a sheet when the parts are cut by guillotine cutting. The systems consist of two parts, genetic algorithms and layout algorithms. The layout algorithms arrange the rectangular parts on the sheet to be able to apply guillotine cutting, while the genetic algorithms optimize the equation expressing the layout of parts so as to make the required sheet length minimum by genetic operations.

1 Introduction

We are studying the application of genetic algorithms(GAs) to two dimensional optimal cutting problems and have proposed the optimal method of cutting parts of various shapes from a sheet in the previous paper [Ono, 1997]. The guillotine cutting problem we are presenting here is another two dimensional optimal cutting problem. In the guillotine cutting, a sheet is always cut from edge to edge, so there are restrictions of the arrangement of parts on the sheet. Until now the optimal guillotine cutting problem has been studied by many researchers with various methods, such as heuristic approach [Ghandforoush, 1992], [Chauny, 1991], AND/OR-graph approach [Morabito, 1996], O(m^3) approximation algorithms [MacLeod, 1993], tree-search algorithms [Christofides, 1995] and Wang's algorithms [Vasko, 1989].

In this paper we are proposing a new method using the GAs. The purpose of our method is to solve the problem of how to cut a fixed number of rectangular parts of various sizes from a sheet by the guillotine cutting in order to achieve the minimum amount of waste.

In the GAs genes are designed to express the equation that determines the arrangement of rectangular parts on a sheet by using two operators : horizontal and vertical arrangement operators, and part numbers assigned to all of the parts consecutively. According to the equation, all the rectangular parts are integrated recursively until finally one large rectangle is obtained, which includes all of the parts. This integration is carried out by the specially designed layout algorithms using the arrangement equation given by the GAs and returns the required length of the sheet to the GAs. In the GAs, by minimizing the length of this integrated rectangle through modifying the equation expressed by the genes, the optimal layout of parts for cutting is gained. By applying this method to the case where the optimal arrangement has been known previously, we have confirmed that the optimal solution is obtained.

2 Basic principle

2.1 Method of part arranging

To solve the optimal arrangement of rectangles, there are two approaches : top-down and bottom-up approaches. In the top-down approach, starting from an original sheet, the cutting proceeds until each rectangle is obtained by combining horizontal and vertical guillotine cutting. In the bottom-up approach, starting from each rectangle, every rectangle is combined horizontally or vertically so as to be able to do the guillotine cutting. In this paper the latter method is adopted, because it is more suitable to our problem.

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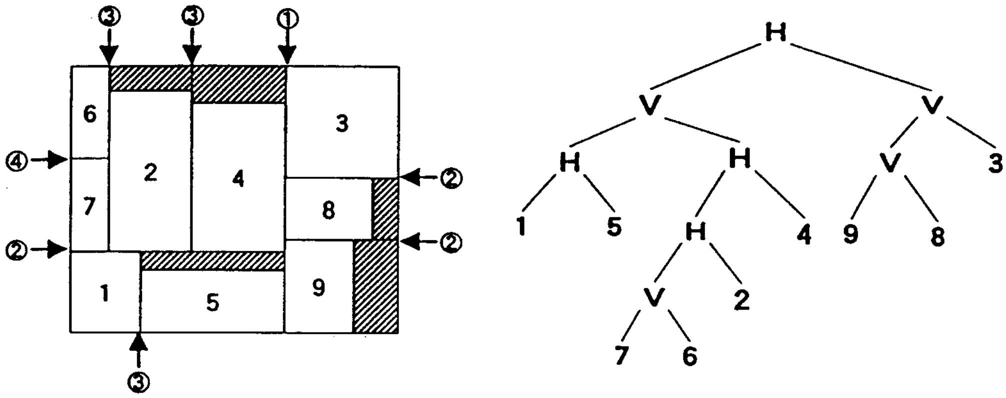


a) Layout of rectangles b) Tree representation

Figure 1: Layout of rectangle parts for guillotine cutting

To set up each part into one large rectangle, the following method is adopted. A pair of rectangles are integrated into a rectangle large enough to cover these rectangles, by arranging horizontally or vertically each other. By applying this method to all of the rectangles recursively, finally one large rectangle is obtained.

2.2 Genetic representation and interpretation

Since the genetic representation should correspond to the arrangement of parts on a sheet without any ambiguity, we have adopted the equation of Inverse Polish Notation using two binomial operators : H- and V-operators. The H-operator takes two rectangle as arguments and produces the minimum rectangle to cover these two rectangles, arranging them side by side horizontally. For example, the equation "1 5 H" means that No.5 rectangle is arranged at the right side of No.1 rectangle and the return value from this equation is the identification number of the rectangle of minimum size to cover these two rectangles. Therefore the horizontal length L_h and vertical length W_h of the integrated rectangle are respectively given by

$$L_h = L_1 + L_5\,,\quad W_h = \max(W_1,\ W_5)\,, \tag{1}$$

where, L_1 and L_5 are the horizontal lengths of these small rectangles and W_1 and W_5 are the vertical lengths. The function max(*, *) is used to calculate the maximum of two values.

The V-operator is similar to the H-operator, except that the two rectangles are arranged vertically. Therefore an equation "1 5 V" indicates that No.5 rectangle is located above No.1 rectangle and the dimensions of the integrated rectangle are determined by

$$L_v = \max(L_1,\ L_5)\,,\quad W_v = W_1 + W_5\,. \tag{2}$$

Let's consider the following equation expressing a layout of rectangular parts.

1 5 H 7 6 V 2 H 4 H V 9 8 V 3 V H

The actual arrangement of the parts on a sheet is shown in Fig. 1, in which the numbers in circles indicate the order of cutting and certainly the guillotine cutting is possible. From the figure we can easily understand that this tree representation also shows how the guillotine cutting is done. Tracing the tree downwards from the root of the tree, we can get the sequence of the guillotine cutting to obtain the required rectangle parts from a sheet.

Here, we will investigate the number of operators necessary for the arrangement and the restriction of their positions in the equation. Let the number of rectangle parts be N_p and the number of operators N_o, then the following relation should hold, because the binomial equation is adopted.

$$N_o = N_p - 1\,. \tag{3}$$

One of the merits brought about by the adoption of the Inverse Polish Notation is that no parenthesis is required to express its function accurately without any ambiguity. Therefore, the length of the equation, that is, the length of the genes N_g is kept constant at

$$N_g = N_p + N_o = 2\,N_p - 1\,, \tag{4}$$

regardless of the arrangement. This is favorable to the design of the genes operators.

Another matter to consider is the restriction to the positions of the part numbers and operators. Let's take any one operator and let the number of the part numbers located at the left side of this operator be n_p and also the number of the operators including itself be n_o , then the following relation should hold.

$$1 \leq n_o \leq n_p - 1\,. \tag{5}$$

As long as the above conditions are satisfied, any expression is allowable and the arrangement is determined definitely. Since these restrictions are irrelevant to the kind of operators, we can make every possible arrangements by selecting proper operators from H- and V-operators, and changing their positions in the equation under the above restrictions.

3 System configuration

The systems consist of two parts, the GAs and the GCLAs(Guillotine Cut Layout Algorithms). The GCLAs arrange rectangle parts on a sheet according to the equation given by the GAs, calculate the necessary length of the sheet and return it to the GAs, while the GAs modify the layout equation to make the required length of the sheet minimum.

3.1 Layout algorithms

The method to interpret the genes and set up the part arrangement can be realized by using the idea of a stack machine. While the equation expressed by the genes is being interpreted from left to right, the following procedures are performed :

1. If it is a part number, then push it onto a stack.
2. If it is either an H- or V-operator, then pull two entries out of the stack, make a integrated rectangle as explained before and push the newly registered part number of this integrated rectangle into the stack.
3. When all the elements of the equation are interpreted, there remains only one entry in the stack, which is the part number of the integrated rectangle of all the parts and the interpretation come to an end. The horizontal length of this rectangle is the required length of the sheet.
4. Return to Step 1.

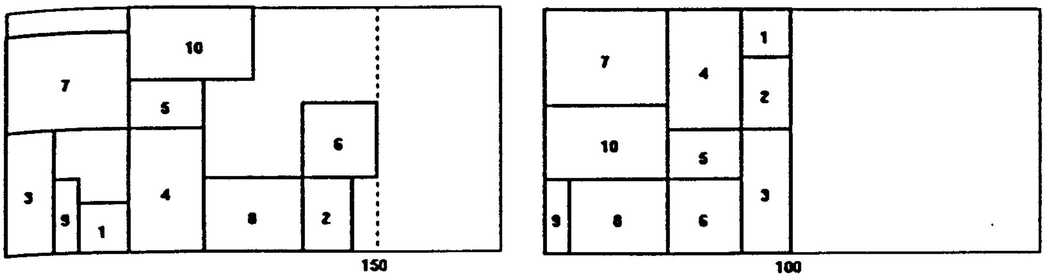
In the midst of the above procedures, the vertical length of the integrated rectangle might become larger than the sheet width. Since this can't be allowed, remedy algorithms are installed. Whenever this has happened, the current operator is changed to the other type, that is, from V-operator to H-operator and this modification is fed back to the GAs with the expression of the genes. As this phenomenon occurs only at the V-operator, no modification is required for the H-operator.

3.2 Genetic algorithms

The GAs should handle the genes expressed by the combination of the part numbers and the operators. The part numbers are treated as an order type in the genetic algorithms, while the operators are a binary bit type, because they take ether one of two states, H- or V-operator. Considering this, we have adopted mixed genetic operations. The genes are separated into two groups, that is, a part number group and an operator group by using a mask. After necessary genetic operations are performed for each group, two groups are again merged into one group of genes and mixed genetic operations are applied.

(1) Crossover

To the part number group, the crossover operator for the order type GAs is applied. The comparison of performance among various methods , such as the PMX(Partial-mapped crossover), OX(Order crossover) and CX(Cyclic crossover) [Michalewicz, 1994] were made and finally the PMX crossover was adopted, because of its superior convergence characteristics.



a) Initial generation b) 36th generation

Figure 2: Best layout of parts gained by simulation

(1) Mutation

The mutation adopted here consists of swapping between two elements in the genes and change-over to the other type of the operators. Let's pick up two elements of the genes at random and name them p_1 and p_2 respectively, where p_1 is nearer to the top of the genes than p_2. According to the kind of elements, suitable type of swapping is selected as follows. When both p_1 and p_2 are part numbers, there is no problem to do mutation, because it is a swapping between part numbers. If p_1 is a part number and p_2 is a operator, a swapping is done only when the requirements for the position of operators are fulfilled as explained. After the operator p_2 is swapped with the part number p_1, the restriction defined by the equation (5) should hold, threfore the following relation should be satisfied to all of the operators existing between p_1 and p_2.

$$n_o \leq n_p - 3 \,. \tag{6}$$

In the other case, that is, p_1 is a operator and p_2 is a part number or a operator, there is no problem at all in doing a swapping.

Other than the above, the mutation of H- or V-operator itself is also possible, that is, the switch-over between H-operator and V-operator. By combining the crossover and mutation explained above, all possible patterns of layout of parts can be possible.

4 Results of simulation studies

To confirm that the performance of the proposed method meets our requirements, we have conducted simulations to the case where the optimal solution is previously known. As a software workbench of the GAs, the Genitor [Michalewicz, 1994] is used, after the necessary modifications to genetic operators including crossover and mutation, and the addition of the layout algorithms have been done. The number of parts to arrange is ten; the number of populations is set at 500. Three kinds of crossover operators, such as the CX, OX and PMX are tested and compared with each other. The PMX showed the superior performance and have been adopted finally.

Fig. 2 shows one example of the layouts of the parts gained at the initial state which is made at random and the 36th generation, which is the optimal solution.

5 Extension to staged cutting

In the guillotine cutting, each time the cutting direction is changed, for example, from horizontal to vertical, the sheets to be cut should be turned by 90 degrees for the next cutting. Since this adds extra work to the job, it is preferable to reduce the number of turns to as few as possible. The least number of turns is two, that is, each one for horizontal and vertical cutting, as shown in Fig. 3 and such a cutting is referred to as two-staged cutting. Actually, one extra cutting is required for some parts to adjust the horizontal length, but one turn for this cutting is excluded in this calculation. This cutting is expressed by the following equation.

$$1\ 2\ 3\ V\ V\ 4\ 5\ 6\ V\ V\ 7\ 8\ 9\ V\ V\ H\ H$$

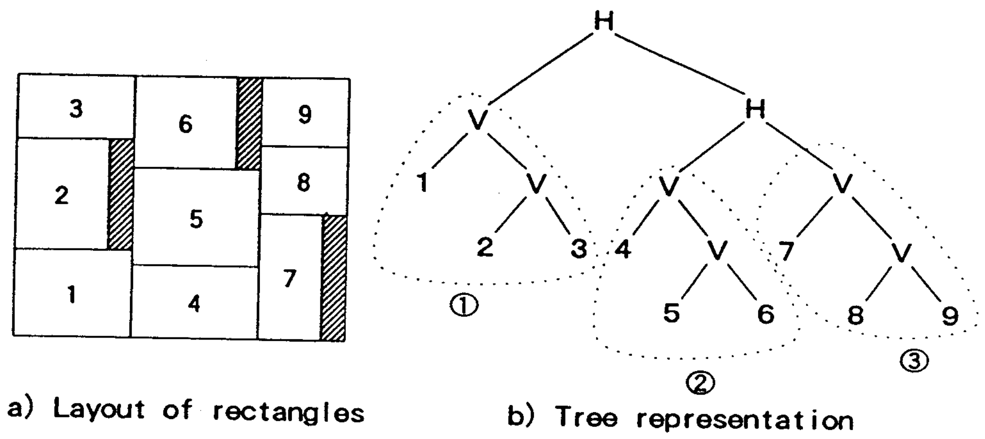


Figure 3: Layout for two staged guillotine cutting

As is understandable from the figure and equation, one of the conditions is that all the H-operators are located at the right side of all the V-operators in the equation. By adding this restriction to the genetic operation, the two staged cutting will be done. Three staged cutting or more will be possible by applying similar conditions.

6 Conclusions

We have explained our new method of guillotine cutting using genetic algorithms. By using the genes defining the layout of rectangles on a sheet by the equation of Inverse Polish Notation with two operators, horizontal and vertical arranging operators, this can be done with the help of layout algorithms. By simulation studies the performance is confirmed. This method is also applicable to a staged guillotine cutting, such as a two-staged cutting or more.

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