Optimizing Two-dimensional Guillotine Cut by Genetic Algorithms

Toshihiko Ono
Dept. of Computer Science and Engineering
Fukuoka Institute of Technology
3-30-1 Wajiro-higashi, Higashi-ku, Fukuoka, 811-0295, Japan
e-mail: ono@cs.fit.ac.jp, http://www.fit.ac.jp/˜ ono/

Abstract

A method of optimizing the layout of rectangular parts on a sheet is proposed, when a guillotine cut is applied. The method consists of two parts, genetic algorithms and layout algorithms. By the layout algorithms, the rectangular parts to be cut are arranged on the sheet so that the guillotine cut is possible, while, by the genetic algorithms, the layout of parts is optimized to make the required sheet length minimum.

1 Introduction

When several rectangular parts are cut from a sheet of glass, it should be always cut from edge to edge, which is referred to as a guillotine cut, and the same method is applied when a sheet of metal is cut to rectangular parts. In a sheet cutting process, where a required number of rectangular sheets of various sizes are cut from an original sheet, they should be arranged on the sheet to make the guillotine cut possible as well as the scrap produced minimal to attain high productivity. The problem taken up here is how to arrange rectangular parts on a sheet to satisfy these two requirements at the same time.

Until now optimal guillotine cutting problems have been studied by many researchers using various methods, such as heuristic approach [1], [3], AND/OR-graph approach [6], O(m^3) approximation algorithms [4], tree-search algorithms [2] and Wang's algorithms [9].

In this paper we are proposing a new method using the genetic algoritms (GAs). The purpose of our method is to solve the problem of how to cut a required number of rectangular parts of various given sizes from an original sheet by the guillotine cut in order to achieve the minimum amount of waste.

In our method, genes are designed to express an equation to determine the arrangement of rectangular parts on a sheet, which consists of two kinds of operators for horizontal and vertical arrangement, and sequential part numbers assigned to all of the parts. According to the equation, all the rectangular parts are integrated recursively until finally one large rectangular sheet including all of the parts is obtained. This integration is carried out by layout algorithms, referring to the arrangement equation given by the GAs as genes, and returns the required length of the sheet to the GAs. On the other hand, in the GAs the genes expressing the arrangement is modified by genetic operation to make the required length of the integrated rectangle as minimal as possible. By combining these two operations, the optimal

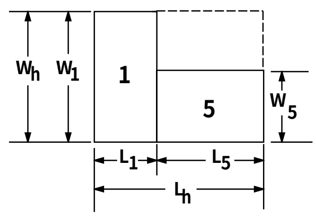


Figure 1: Horizontal integration of rectangles

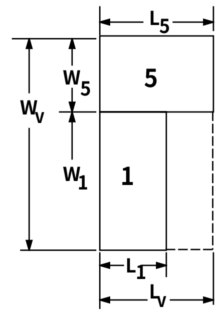


Figure 2: Vertical integration of rectangles

layout of parts is gained. It was confirmed by simulation studies that the optimal solution is obtained by this method, to the case where the optimal arrangement is known.

2 Basic principle

2.1 Method of part arranging

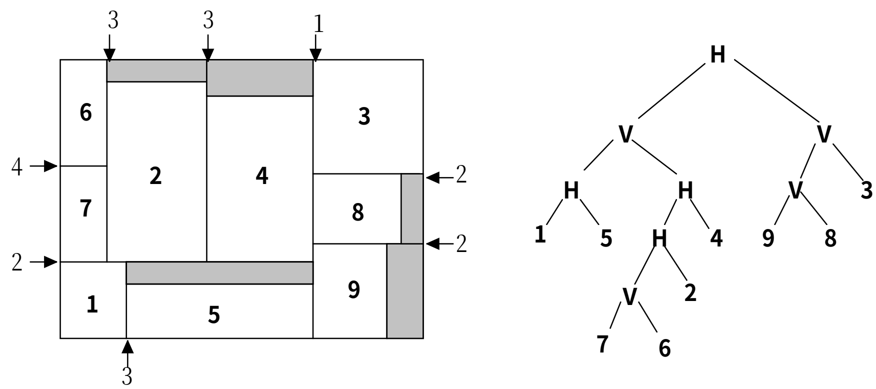
To solve the optimal arrangement of rectangles on a sheet, there are two approaches : top-down and bottom-up approach. In the top-down approach, the arrangement is determined by referring to an actual cutting process. Starting from an original sheet, the cutting process proceeds until all of the required number and shape of rectangle parts are obtained, by selecting a proper cutting method from horizontal and vertical guillotine cut to each cut.

In the bottom-up approach, starting from each required rectangle, two (or more) rectangles are combined horizontally or vertically to produce the minimum rectangle able to cover those rectangles recursively, until one rectangular sheet is gained finally. Apparently the sheet thus gained can be cut to each part by the guillotine cut. Having compared these two approaches, we have decided to adopt the latter bottom-up approch, because it is more suitable to our problem.

To set up all the parts into one large rectangle, the following method is adopted here. A pair of rectangles are integrated into the minimum rectangle large enough to cover these two rectangles, by arranging horizontally or vertically each other. By applying this method to all of the rectangles and integrated rectangles thus gained recursively, finally one large rectangle is obtained.

2.2 Genetic representation and interpretation

To make the genes represent the arrangement of parts on a sheet without any ambiguity, we have adopted the equation of Inverse Polish Notation using two binomial operators : H- and V-operator. The H-operator takes two rectangles as arguments and produces the minimum rectangle able to cover these two rectangles, arranging them side by side horizontally. For example, the equation "1 5 H" means that No.5 rectangle is arranged at the right side of No.1 rectangle as shown in Fig. 1 and it return the identification number of this integrated rectangle having minimum size to cover these two rectangles.



a) Layout of rectangles b) Tree representation

Figure 3: Layout of rectangle parts for guillotine cut

Therefore the horizontal length L_h and vertical length W_h of the integrated rectangle are respectively given by

$$L_h = L_1 + L_5 \ , \ \ W_h = \max(W_1, \ W_5) \ , \tag{1}$$

where, L_1 and L_5 are the horizontal lengths of these two small rectangles and W_1 and W_5 are their vertical lengths. The function $\max(*, *)$ is used to calculate the maximum of two values.

The V-operator is similar to the H-operator, except that the two rectangles are arranged vertically. Therefore an equation "1 5 V" indicates that No.5 rectangle is located above No.1 rectangle as shown in Fig. 2 and the dimensions of the integrated rectangle are determined by

$$L_v = \max(L_1, \ L_5) \ , \ \ W_v = W_1 + W_5 \ . \tag{2}$$

Let's consider the following equation expressing a layout of rectangular parts as an example.

$$1 \ 5 \ H \ 7 \ 6 \ V \ 2 \ H \ 4 \ H \ V \ 9 \ 8 \ V \ 3 \ V \ H$$

The arrangement of the parts on a sheet defined by this eqation is shown in Fig. 3, in which numbers with arrows indicate the order of cut, and certainly the guillotine cut is possible. From the figure we can easily understand that this tree representation also shows how the guillotine cut is done. By tracing the tree downwards from the root of the tree to leaves, we can get the sequence of guillotine cut to obtain the required rectangular parts from a sheet.

Here, we will explained some characteristics existing in this equation. First, there are restrictions on the number of operators used for the arranging of sheets and on the positions of these operators in the equation. Let the number of rectangular parts be N_p and the number of operators N_o respectively, then the following relation should hold, because the binomial equation is adopted here.

$$N_o = N_p - 1 \ . \tag{3}$$

Second, by the adoption of the Inverse Polish Notation, no parenthesis is required at all to express its function accurately without any ambiguity and as a result the length of the equation, that is, the length of the genes N_g is kept constant at

$$N_g = N_p + N_o = 2 \ N_p - 1 \ , \tag{4}$$

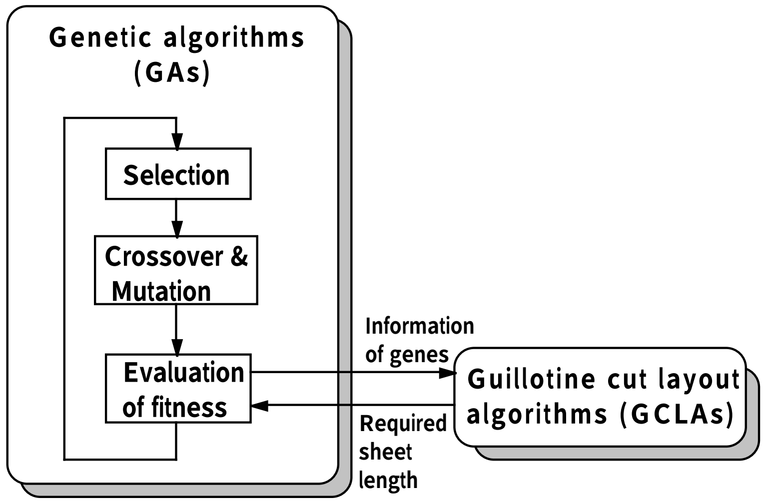


Figure 4: System configuration

regardless of the arrangement of parts, which is favorable to the design of the genetic operators.

Another matter to consider is a restriction to the relative positions of the part numbers and operators. Let's take any one operator; let the number of part numbers located at the left side of this operator be n_p and the number of operators, including itself, at the left side of this operator be n_o respectively, then the following relation should hold.

$$1 \leq n_o \leq n_p - 1 \ . \tag{5}$$

As long as the above-mentioned conditions are maintained, any arrangement of operators and part numbers is acceptable and the corresponding arrangement of rectangular parts is determined definitely.

Since these relations are irrelevant to the kind of operators, every possible arrangements can be obtained by selecting proper operators from H- and V-operator and by changing their positions as well as the order of part numbers in the equation.

3 System configuration

The systems consist of two parts, the GAs and the GCLAs(Guillotine Cut Layout Algorithms) as shown in Fig. 4. The GCLAs arrange rectangular parts on a sheet according to the equation given by the GAs as the genes, calculate the necessary length of the sheet and return it to the GAs. On the other hand, the GAs modify the genes expressing the layout equation to make the required length of the sheet minimum.

3.1 Layout algorithms

The method to interpret the genes and set up the arrangement of parts is realized by using the idea of a stack machine as shown in Fig. 5. While the equation expressed by the genes is being interpreted from left to right, the following procedures are performed :

1. If it is a part number, then push it onto a stack.

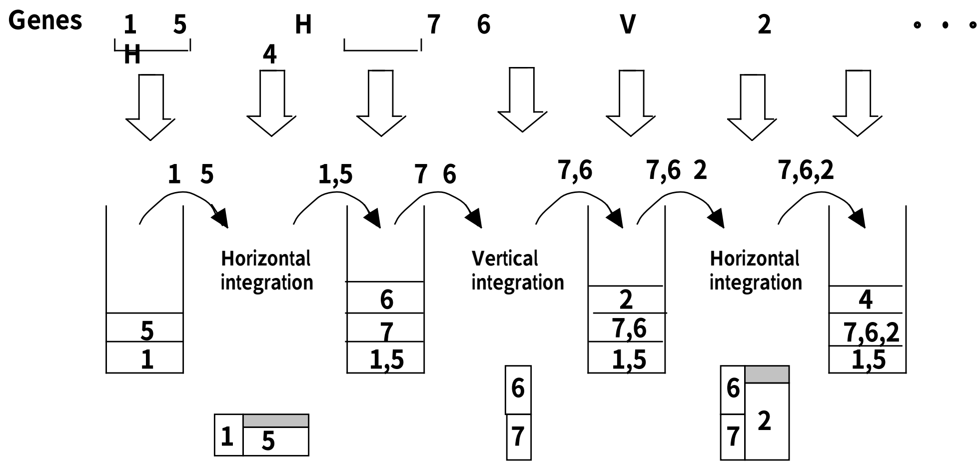


Figure 5: Interpretation of equation

2. If it is either an H- or V-operator, then pull two entries out of the stack, make an integrated rectangle and push the newly registered part number of this integrated rectangle onto the stack.

3. When all the elements of the equation are interpreted, there remains only one entry on the stack, which is the part number of the integrated rectangle of all the parts and the interpretation operation come to an end. The horizontal length of this rectangle is the required length of the sheet.

4. Return to Step 1.

In the midst of these procedures, the vertical length of the integrated rectangle might become larger than the sheet width. Since this can't be allowed, remedy algorithms are installed. Whenever this has happened, the current operator of the equation is changed to the other type, that is, from V-operator to H-operator and this modification is fed back to the GAs in the expression of the genes. No modification is required for the H-operator, because the similar problem doesn't happen in this case.

3.2 Genetic algorithms

Since our GAs have to handle the genes expressed by a combination of part numbers and operators, the genetic operators are different from those used in the ordinary ones. The part numbers are treated as an order type, while the operators as a binary bit type, which take ether one of two states, H- or V-operator. Considering this, we have adopted mixed genetic operations. The genes are separated into two groups, that is, a part number group and an operator group by using a mask as shown in Fig. 6. After necessary genetic operations are performed at each group, two groups are merged into one group of genes again and mixed genetic operations to both of the operators and the part numbers are applied.

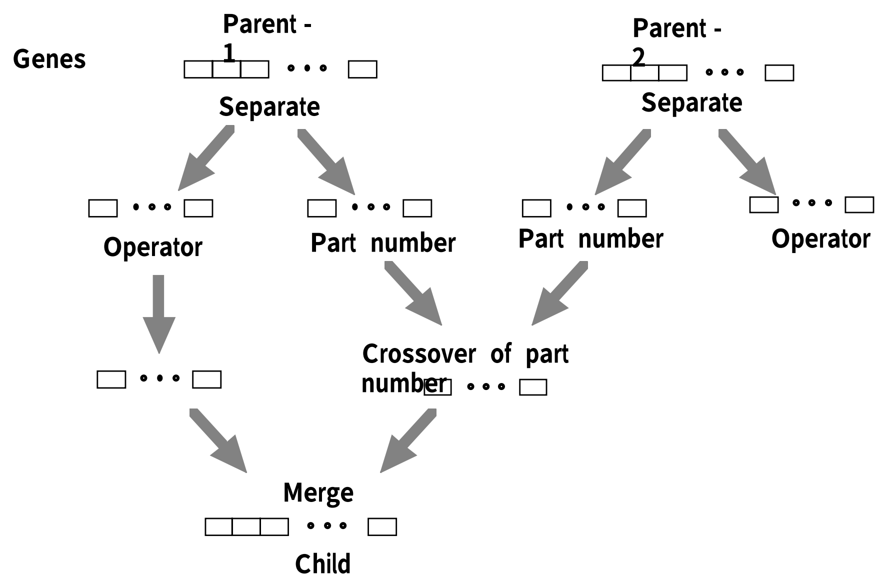


Figure 6: Separation of genes

(1) Crossover

To the group of part numbers, the crossover of the order type GAs are applied. Since there are many kinds of crossover method, after the comparison of performance among various methods, such as the PMX(Partially-mapped crossover), OX(Order crossover) and CX(Cyclic crossover)[5], were made, finally the PMX crossover was adopted, because of its superior convergence characteristics.

(1) Mutation

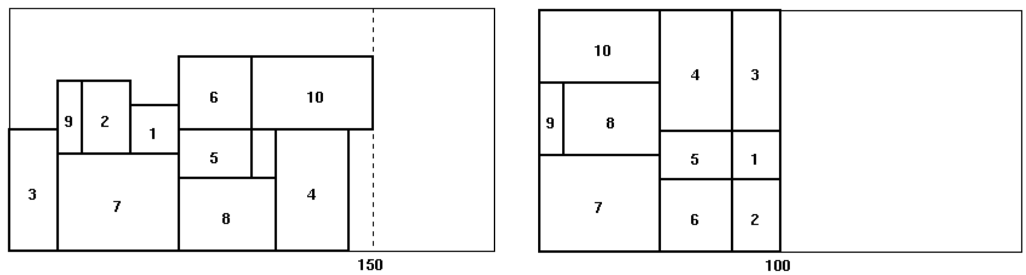
The mutation adopted here consists of a swapping between two elements in the genens, including both of the operators and part numbers, and a change-over to the other type of the operators. Let's pick up two elements of the genes at random and name them p_1 and p_2 respectively, where p_1 is located at the left-hand side of p_2. According to the kind of elements, a suitable type of swapping is selected as follows. When both p_1 and p_2 are part numbers, a swapping between them is allowable. If p_1 is a part number and p_2 is a operator, a swapping is done only when the requirement for the position of operators is fulfilled as followed. After the operator p_2 is swapped with the part number p_1, since the restriction defined by the equation (5) should hold, the following relation should be satisfied to all of the operators existing between p_1 and p_2.

$$n_o \leq n_p - 3 \, . \tag{6}$$

In other case, where p_1 is a operator, there is no problem at all in doing a swapping, regardless of the kind of p_2.

Other than the above procedure, the mutation of H- or V-operator itself, that is, the switch-over between these operators is possible.

By combining the crossover and mutation operations explained above, all the possible patterns of layout of parts will be realized.



a) initial generation b) 18th generation

Figure 7: Layout of rectangular parts gained at simulation

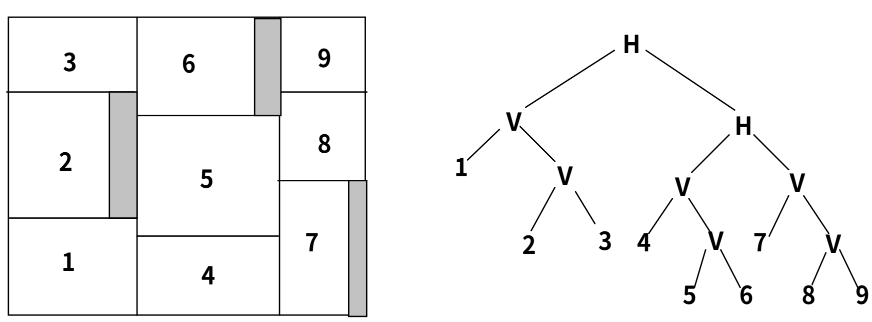
4 Results of simulation studies

To confirm the performance of the proposed method, we have conducted simulation studies to the case where the optimal solution is known. As a software workbench of the GAs, the Genitor [5] was used, after necessary modifications were made to genetic operators including crossover and mutation, and the layout algorithms were added. The number of rectangular parts to arrange is ten; the number of populations is set at 500. As a crossover the PMX was adopted, after various crossover methods were tested and compared on their performances.

Fig. 7 shows one example of the layouts of the parts, gained at the initial state of random arrangement, and the 18th generation, when the optimal solution is gained.

5 Extension to staged guillotine cut

In the guillotine cut, each time the cutting direction is changed, for example, from horizontal to vertical or vice versa, the sheets to be cut should be turned by 90 degrees for the next cut. Since this adds extra work to the job, it is preferable to reduce the number of turns as much as possible. The least number of turns is two, that is, each one for horizontal and vertical cut, as shown in Fig. 8 as an example and such a cutting is referred to as a two-staged guillotine cut. Actually, one additional cutting may be needed for some parts to adjust the horizontal length to the required one, this number is excluded. The cutting method shown in the figure



a) Layout of rectangles b) Tree representation

Figure 8: Layout for two-staged guillotine cut

is expressed by the following equation.

$$1\ 2\ 3\ V\ V\ 4\ 5\ 6\ V\ V\ 7\ 8\ 9\ V\ V\ H\ H$$

One of the conditions on the arrangement equation, under which the two staged guillotine cut is possible, is that all the H-operators are located at the right side of all the V-operators. By adding this restriction to the genetic operation, the two-staged guillotine cut will be realized.

6 Conclusions

We have explained our new method of optimizing guillotine cut by the genetic algorithms, in which the genes define the layout of rectangular parts on a sheet by the equation of Inverse Polish Notation with horizontal and vertical arrangement operators, in cooperation with the layout algorithms to arrange the parts on the sheet according to the genes. By simulation studies, it is confirmed that this method meets our expectations. This method is also applicable to a staged guillotine cut, such as a two-staged guillotine cut.

This study has been done with our master student, Mr. Ikeda, and I am grateful for his endeavor.

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